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THE NATURAL-CONVECTION BOUNDARY LAYER IN RAREFIED GAS

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Аннотация—Приведены данные оптического и термометрического исследования теплового пограничного слоя при свободной конвекции около вертикальной пластины в вакууме.

NOMENCLATURE

- x, vertical co-ordinate along the plate;
- y, co-ordinate normal to the plate;
- z, horizontal co-ordinate along the plate;
- l, characteristic dimension;
- δ, boundary-layer thickness;
- P, pressure [mmHg];
- θ, dimensionless temperature;
- n, refractive index;
- T, absolute temperature;
- Δt, temperature difference;
- φ, ray reflection angle;
- S, path length of rays along the heated plate;
- Δf, linear displacement of the ray in the focus,
- f, focal distance of the instrument (1917 mm);
- $\frac{dt}{dy}$, temperature gradient normal to the plate;
- Gr, Grashof number;
- Kn, Knudsen number;
- Pr, Prandtl number.

AT LOW pressures, the effect of convective heat transfer is reduced, and heat conduction becomes a dominant mode of heat transfer. Consequently, the effect of the shape of the body from which heat is transferred to the environment increases.

Another peculiarity of low-pressure heat transfer is the influence of temperature discontinuity and slip which specify rarefaction. This takes place at $Kn > 0.02$ as has been experimentally shown in references [2, 8].

The Grashof number is expressed in terms of Kn as

$$Gr = 0.054 \frac{\Delta t}{T^2} \frac{l}{Kn^2} \qquad (1)$$

Bearing in mind that the process occurs in an infinite volume, we shall approximately estimate the dimensions of the environment at which free convection may still affect heat transfer. For the heat-transfer conditions of interest, the Grashof number based on the characteristic dimension of the environment $Gr_0 > 1000$ [1]

$$\frac{Gr_0}{Gr} = \left[\frac{l_0}{l}\right]^3$$

Substitution of the value of Gr from equation (1) at $Kn = 0.02$ and the inequality $Gr_0 > 1000$ yields

$$l_0 > 1.94 \sqrt[3]{\left[\frac{l^2 T^2}{\Delta t}\right]} \qquad (2)$$

It follows from equation (2) for particular temperature conditions and dimensions of the environment that with simultaneous effect of temperature discontinuity, free convection may affect heat transfer only in the case of bodies with small characteristic dimensions.

In the present work, the experimental data on a boundary layer near a vertical plate at low pressure are given when temperature discontinuity still has no effect on the process. For a semi-infinite plate Eckert [3] has obtained characteristics of a laminar boundary layer from integral relations, and Ostrach [4] from asymptotic layer theory.

Eckert's relation for the boundary-layer thickness in the air

$$\frac{\delta}{x} = \frac{5{\cdot}28}{Gr_x^{\frac{1}{4}}} \tag{3}$$

may be used as the first approximation for estimation of the region of thermal disturbance at low pressure. The relation yields

$$\delta \sim \frac{1}{\sqrt{P}} \tag{4}$$

It is obvious that when pressure is reduced by several orders of magnitude, the boundary-layer thickness must increase and under certain conditions becomes larger than the characteristic dimensions of the body. In this case heat transfer may not be governed by the laws obtained for the conditions when it is possible to reduce the Navier-Stokes equations to the Prandtl equations.

Experimental investigation of the characteristics of the boundary layer are of great significance for the analysis of heat transfer. Application of the schlieren method for this purpose is of particular interest, as the possibilities of optical techniques in research on free convection at low pressure have not been completely revealed so far.

Experiments were carried out in a vacuum apparatus which is described in detail in reference [2]. The shadow instrument IAB-451 was used for optical measurement. An assembly of seven nichrome–constantan thermocouples with electrodes 0·1 mm in thickness was mounted to register temperature in the gas. The work was concerned with a thermal boundary layer near a plate 119 cm high, 210 mm long and 1·75 mm thick. The plate was parallel to the ray path in the instrument. The plate was constructed of two copper plates 0·5 mm in thickness with a plane heater of stainless steel 0·2 mm thick inserted between the copper plates in mica insulation. The copper plates were riveted to each other. Nichrome–constantan thermocouples with electrodes 0·1 mm thick were let into the external surfaces of the plates.

Calorimetric, optical and thermometric observations were made simultaneously in the pressure range from 760 to 10 mm Hg. Particular care was taken in obtaining the ray deflection pattern in the thermal boundary layer at low pressures. A similar work at atmospheric pressure was carried out by Kozlova [5].

When the schlieren method is used for investigation of the thermal boundary layer at constant pressure, the following expression relating the refraction index and air parameters is used

$$\frac{1}{n}\frac{\mathrm{d}n}{\mathrm{d}T} = -\frac{0{\cdot}08}{T^2}\frac{P}{760} \tag{5}$$

This is obtained from the relation of the refraction index and the gas density based on the density at normal conditions (pressure of 760 mmHg and temperature of 0°C) [6].

The deflection angle in the y-direction normal to the z-direction of the ray is

$$\varphi_y = \frac{1}{n}\int_{z_1}^{z_2} \frac{\mathrm{d}n}{\mathrm{d}y}\,\mathrm{d}z \tag{6}$$

Application of formulae (5) and (6) in the simplest case when the temperature gradient along the ray path is constant yields

$$\varphi_y = -1{\cdot}053 \,.\, 10^{-4} \,.\, S \,.\, \frac{P}{T^2} \cdot \frac{\mathrm{d}T}{\mathrm{d}y} \tag{7}$$

Linear ray deflection in the boundary layer near a plate is

$$\Delta y = -0{\cdot}526 \,.\, 10^{-4} \,.\, S^2 \frac{P}{T^2} \cdot \frac{\mathrm{d}T}{\mathrm{d}y} \tag{8}$$

Linear ray deflection in the focus of the instru-

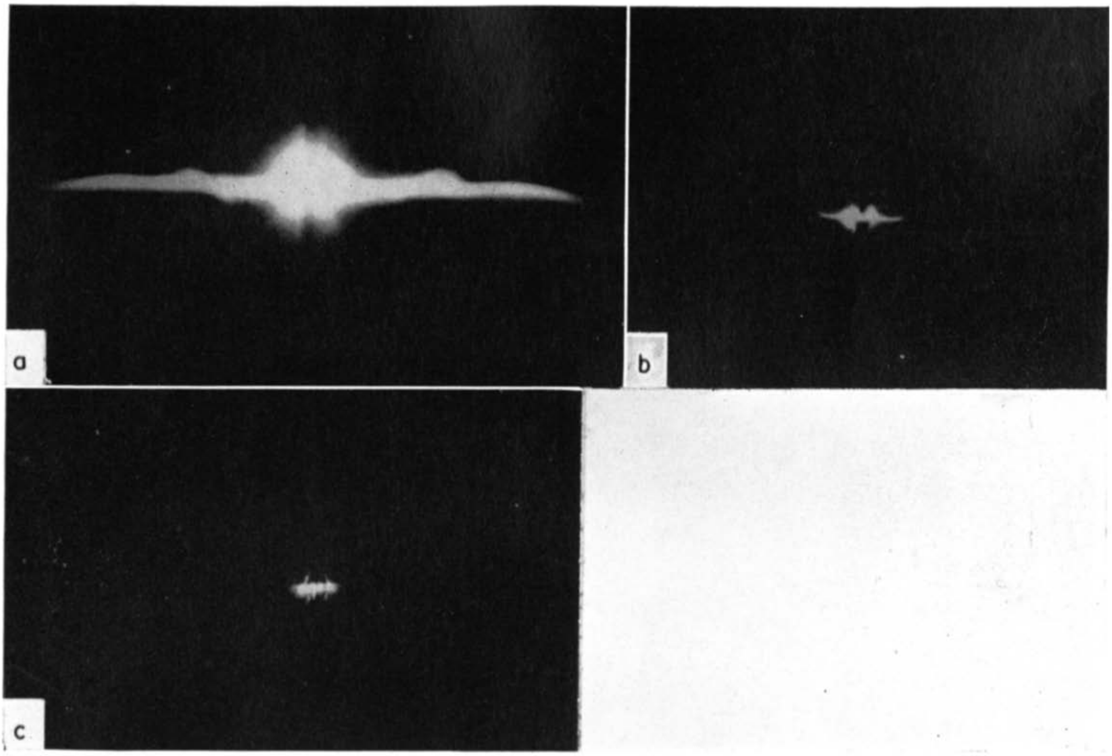

FIG. 1. Image of light source in focus (4 times enlarged).

(a) P = 755 mmHg; Δt = 40° degC; slot 0·3 mm; wire 0·16 mm.
(b) P = 200 mmHg; Δt = 80° degC; slot 0·1 mm; wire 0·16 mm.
(c) P = 40 mmHg; Δt = 152° degC; slot 0·1 mm; wire 0·16 mm.

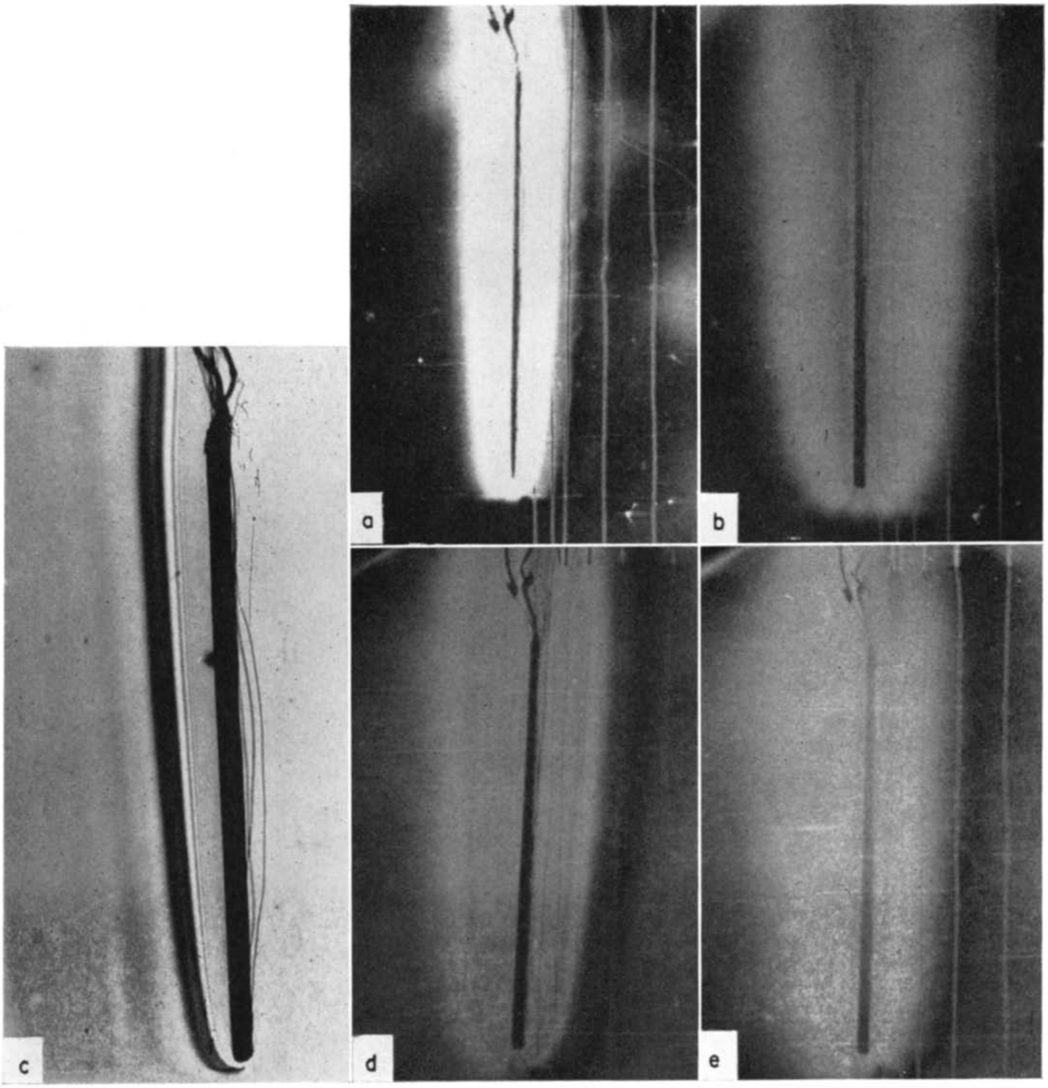

FIG. 2. (a, b)—Schlieren picture of heated vertical plate; (c, d, e)— schlieren-picture with a wire in focal plane.

(a) $P = 755$ mmHg; $\Delta t = 92{\cdot}3°$ degC.
(b) $P = 50$ mmHg; $\Delta t = 205{\cdot}4°$ degC.
(c) $P = 755$ mmHg; $\Delta t = 40°$ degC; focal deflection $\Delta t = 0{\cdot}07$ mmHg.
(d) $P = 105$ mmHg; $\Delta t = 105{\cdot}5°$ degC; $\Delta t = 0{\cdot}03$ mm.
(e) $P = 53$ mmHg; $\Delta t = 109°$ degC $\Delta t = 0{\cdot}04$ mm.

ment IAB-451 is

$$\Delta f = 1{\cdot}053 \,.\, 10^{-4} \,.\, f \,.\, S \,.\, \frac{P}{T^2}\frac{\mathrm{d}T}{\mathrm{d}y} \qquad (9)$$

The value of deflection allows determination of the temperature field and calculation of the thermal boundary layer. For this aim at different pressures:

(1) The light source image was photographed in the focal plane of the instrument;

(2) the boundary-layer image was photographed when the thread (optical knife) was in the focus;

(3) the temperature field in the boundary layer was investigated by the method of a focused thread.

The image of the light source in the focus is broad because of the ray deflection in the thermal boundary layer. In Fig. 1 one can see light protrusions in the form of vertically symmetrical light "wings". The value of the largest protrusion corresponds to the greatest temperature gradient (in the present case at the lower edge of the plate).

Application of this method for the quantitative analysis is restricted since

(a) this method allows estimation only of the maximum temperature gradient;

(b) absence of a clear boundary of the protrusions makes measurement difficult;

(c) at low pressures the protrusions are very small: they are commensurable with the dimensions of the image of the light source or smaller than the image.

The photographs in Fig. 2(a, b) for the case of the thread placed exactly in the focus show quantitative changes of the boundary layer near the plate at low pressure. The object (a light source) is screened by the thread. Approximate estimation of the boundary-layer thickness according to the "visible" boundaries does not contradict the theoretical assumption of the boundary layer thickness

$$\delta \sim \frac{1}{\sqrt{P}}.$$

For quantitative studies, the thread method is more adequate. The thread is fixed in the focal plane of the instrument. It is strictly parallel to the edge of the rectangular image of the light source (a slot). When the thread is placed in the centre of the image of the light source, it serves as the optical knife to cut off the rays with the minimum deflection, and on the screen we observe a schlieren picture of the boundary layer [Fig. 2(a, b)].

Quantitative data may be obtained by placing the thread at various distances from the focus. On the illuminated field at the heated plate one can see a shadow of the thread as a broad border. Its position in the boundary layer determines the region with the same temperature gradient normal to the plate surface. Fig. 2(c, d, e) shows "visible" boundaries of the thermal boundary layer at different pressures.

Decrease of pressure leads to a noticeable growth of the border width, that shows smoother variation of the temperature. To plot the temperature distribution, measurements were made of the position of the thread relative to the focal point to within $\pm 0{\cdot}01$ mm and position of the thread shadow on the film with the image of the plate by means of the measuring microscope MIR-12. At low pressures, the accuracy is not very high since there is no clear definition of the shadow. For example, at a pressure of 40 mmHg, the width of the thread shadow is above 20 mm.

Determination of external boundaries of the boundary layer at low pressure is difficult because of restricted sensitivity of the instrument. The minimum ray deflection which may be recorded is independent of the process conditions. It may be assumed constant for any pressure. Proceeding from equation (7), we may write the equality at two different pressures

$$\frac{P_1}{T_1^2}\left(\frac{\mathrm{d}T}{\mathrm{d}y}\right)_1 = \frac{P_2}{T_2^2}\left(\frac{\mathrm{d}T}{\mathrm{d}y}\right)_2 \qquad (10)$$

Here temperature gradients correspond to the smallest deflection angles which can be measured by the instrument. Bearing in mind that at the borders of the boundary layer $T_1 = T_2$, we obtain

$$\left(\frac{\mathrm{d}T}{\mathrm{d}y}\right)_2 = \frac{P_1}{P_2}\left(\frac{\mathrm{d}T}{\mathrm{d}y}\right)_1 \tag{11}$$

Hence the lowest temperature gradients that can be measured by the instrument increase proportionally to pressure decrease. This explains, for example, the fact that at a temperature difference of 40 degC, when pressure decreases from 760 to 40 mmHg, the image of the boundary layer almost completely disappears.

Thus, Tepler's instrument may be used at low pressures for investigation of the temperature field of the boundary layer within the range of the temperature gradient of interest. Its value in the present experiments was

$$\left(\frac{\mathrm{d}t}{\mathrm{d}y}\right)_{\min} \approx \frac{600}{P}\frac{\mathrm{deg}}{\mathrm{cm}} \tag{12}$$

where P is measured in mmHg.

Boundaries of the temperature layer which are seen on the screen with the image of the plate are determined by the contour with the maximum temperature gradient which can be measured. The value of δ defined above may be either more or less than the distance along the y-axis to the contour with $(\mathrm{d}t/\mathrm{d}y)_{\min}$ depending on the conditions. The boundary-layer thickness found by the optical method is not therefore a physical property of the process as it was assumed in reference [7].

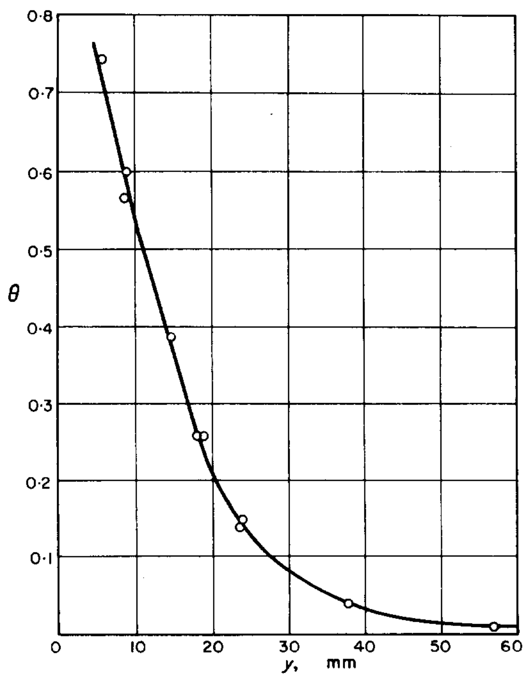

FIG. 3. $Gr_m = 0{\cdot}5 \cdot 10^4$.

In contrast to optical methods, investigation of the thermal boundary layer by means of an assembly of thermocouples still yields satisfactory results at low pressure. The assembly consisted of 7 thermocouples at distances of 6; 9; 15; 24; 38; 57; 91 mm from the plate. Deviations of thermocouple readings from the true ones due to radiant heat transfer are predicted and introduced as a correction factor to the measured values. The correction factor is not very large for thermocouples nearest to the plate and it becomes significant at a temperature difference of above 100 deg (above 1 deg).

To investigate the effect of the temperature factor on the nature of the boundary layer, an experiment was carried out at constant value of $(Gr\ Pr)$ based on the mean temperature of the boundary layer. At a value of $(Gr\ Pr)$ of $0{\cdot}5 \cdot 10^4$ when pressure falling from 166 to 50 mmHg, the temperature difference Δt increased from 5 to 205 deg. Figure 3 shows the dimensionless temperature distribution for the centre of the plate at the above conditions. Experimental points show that the relation between the temperature and the value of $(Gr\ Pr)$ in the pressure range of interest is single-valued.

Determination of the dimensions of the boundary layer involves great errors in measurements of small temperature differences. It is therefore more reasonable to use integral characteristics for estimation of dimensions of the boundary layer. The thermal displacement thickness may be used as such a quantity

$$\delta_T = \int_0^{\delta} \theta\, \mathrm{d}y \tag{13}$$

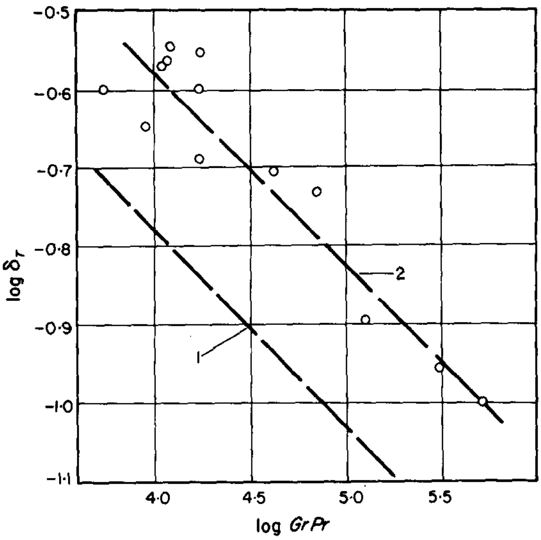

FIG. 4. 1 − $\delta_T = 1{\cdot}64\,(Gr\,Pr)^{-\frac{1}{4}}$;
2 − $\delta_T = 2{\cdot}64\,(Gr\,Pr)^{-\frac{1}{4}}$.

According to Eckert [3] $\theta = (1 + y/\delta)^2$

$$\delta = 3{\cdot}93\,.\,Pr^{-\frac{1}{2}}(0{\cdot}932 + Pr)^{\frac{1}{4}}\,(Gr_x)^{-\frac{1}{4}}\,x^{\frac{1}{4}} \qquad (14)$$

Substitution of the value of θ into (13) yields

$$\delta_T = \frac{\delta}{3} \qquad (15)$$

Figure 4 furnishes experimental values of $\delta_T = (\delta_T/l)$ in the range of $(Gr\,Pr)$ considered. The plot shows certain scatter of points with temperature. They all lie in the average 50 per cent above the relation predicted by formulae (14, 15).

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Abstract—In the paper the results are presented of optical and thermometric investigations of the thermal boundary layer near a vertical plate during free-convection heat transfer to air at low pressure.

Résumé—On présente ici les résultats de recherches optiques et thermométriques dans la couche limite thermique le long d'une plaque verticale au cours du transport de chaleur avec convection libre dans de l'air à faible pression.

Zusammenfassung—In der Arbeit werden die Ergebnisse optischer und thermometrischer Untersuchungen mitgeteilt die an der Grenzschicht einer senkrechten Platte beim Wärmeübergang in freier Konvektion an Luft geringen Druckes durchgeführt wurden.